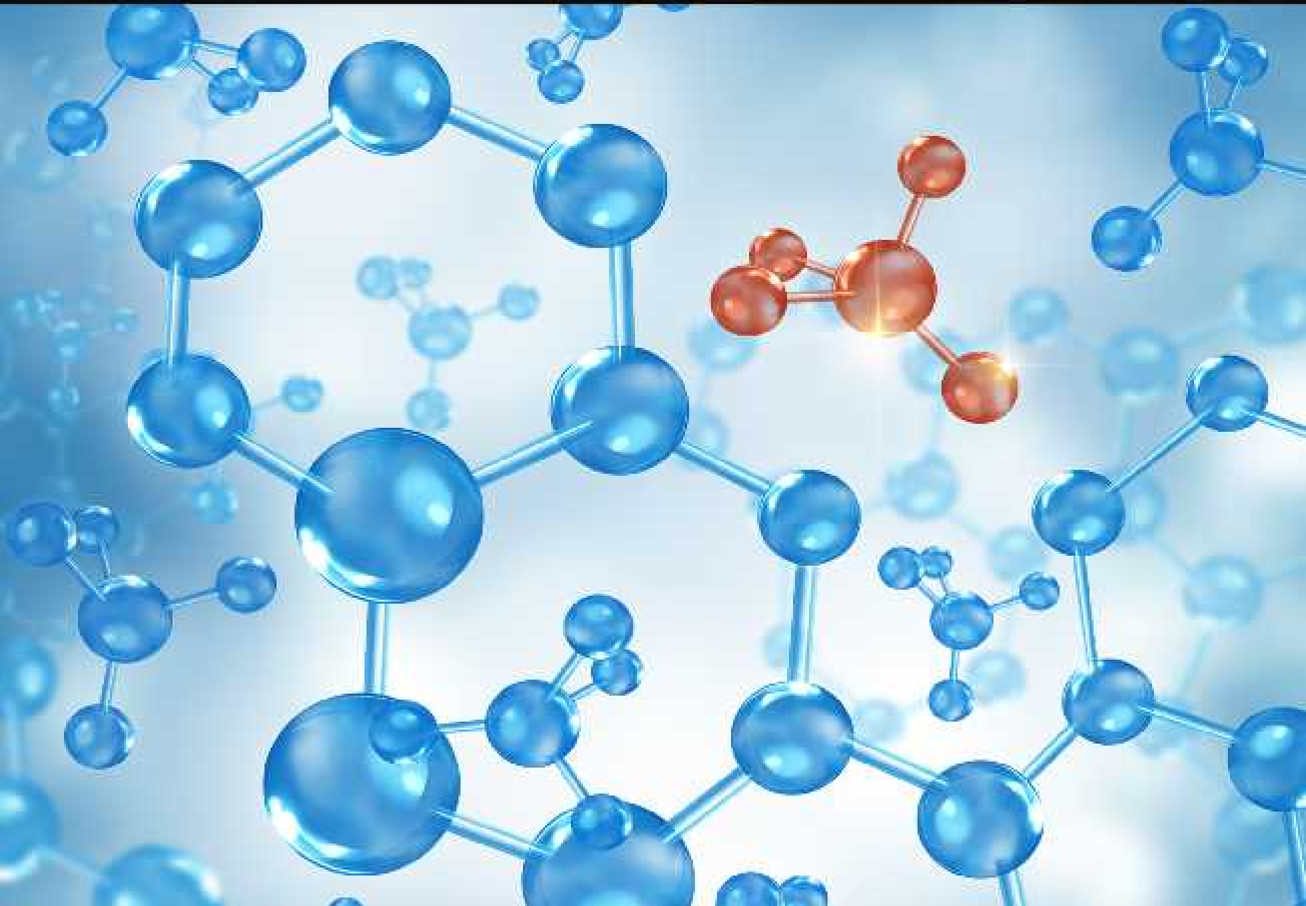
Where the Biological Antimatter is?

Luis Grave de Peralta



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Contents

PREFACE

It is evident that living beings made of matter surround us. These living beings contain atoms with electrons and protons. However, there are not living beings made of antimatter surrounding us. This hypothetical living beings made of antimatter should contain antimatter atoms with positrons and antiprotons. Why does this biological matter-antimatter asymmetry exist? This work presents a surprisingly simple answer to this question. In short, this is a theoretical consequence of the introduction of special relativity in quantum mechanics. In addition, it is necessary to assume that both an electrically charged particle with mass and the corresponding antiparticle could interact electrically with itself. Finally, for breaking the theoretical matter-antimatter symmetry, it is necessary to postulate that a particle electrically interacts with itself differently than the corresponding antiparticle interacts with itself.

The question “where the biological antimatter is?” is not a trivial question. This is because its answer brings transcendental implications for our best physical theories. Fortunately, the answer to this question presented in this book coincides with our everyday experiences. Biological antimatter does not surround us because biological antimatter cannot exist. The author invites the readers to follow him through this exercise of scientific curiosity.

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ABSTRACT

We are surrounded by living beings made of matter. However, no living beings made of antimatter have ever been observed. This looks like a huge wrong prediction of our best theories about the physical fundaments of our world. In this work, the author advances a possible explanation of this mystery. This work discusses how we could explain the everyday experience of the absence of biological antimatter in our world by maintaining the validity of relativistic quantum mechanics but adopting the idea that an electron like a positron could interact electrically with itself. For simplicity, the discussion is based on the solution of the Grave de Peralta equation for the infinity well. This is the simplest model for a spatially localized relativistic quantum particle with mass. A semiquantitative discussion of the consequences of adding the interaction of the quantum particle with itself is presented. The matter-antimatter symmetry is broken by postulating that a particle electrically interacts with itself in a different way than the corresponding antiparticle interacts with itself.

Keywords: Antimatter; biological antimatter; special relativity; quantum mechanics; relativistic quantum mechanics.

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INTRODUCTION

We are surrounded by living beings made of matter. However, there are no known living beings made of antimatter. At first sight, this everyday fact looks so trivial that it doesn't justify any scientific curiosity about it. Unfortunately, there is a deep reason for this apparent lack of scientific curiosity.

The standard model of particle physics is currently considered by most physicists as our best theory about the physical fundaments of our world [1]. In this theory, an antiparticle made of antimatter corresponds to each particle made of matter. The exceptions are some particles that are their own antiparticles. Particles and antiparticles are created from the quantum vacuum in pairs [2]. Moreover, the standard model of particle physics predicts that, besides a few exceptions, there should be a matter-antimatter symmetry in Mother Nature [3-4]. Consequently, a world where antimatter should be as abundant as matter is the big picture of the world predicted by the standard model of particle physics. This contradicts our everyday experiences. We are surrounded by matter, but antimatter is scarce in the known universe.

Moreover, we are surrounded by living beings made of matter. However, no living beings made of antimatter have ever been observed. This looks like a huge wrong prediction of our best theories about the physical fundaments of our world. Unfortunately, the apparent lack of scientific curiosity about the inexistence of biological antimatter may be a subtle effort to hide the shortcomings of our best physical theories.

In this monograph, the author advances a possible explanation of this mystery. Of course, the author’s hypothesis should be a controversial one because it challenges current viewpoints in the standard model of particle physics. Looks like there is no other way of explaining our everyday experiences. The basic idea behind the author’s hypothesis is this: each quantum particle and antiparticle interact with itself.

Why this simple idea is so controversial? Clearly, every macroscopic object surrounding us interacts with itself. For instance, the existence of magma in the interior of our planet is a consequence of the huge gravitational attraction of some parts of the Earth produced on other parts of it. Also, it is well known the existence of quantum objects that interact with themselves. For instance, the Hydrogen atom exists because there is an electromagnetic interaction between the electron and the proton forming it. The difficulty appears when we consider fundamental quantum particles. In the standard model of particle physics, the fundamental quantum particles are literally points with null size; therefore, fundamental particles like electrons do not have parts that could interact with each other.

In the standard model of particle physics, an electron is a mathematical point; therefore, an electron cannot interact with itself. This extremely mathematical idea about the electron is challenged by the author in this monograph. The author’s conceded this is currently a controversial idea, but curiosity is in science’s heart. In this monograph, the author discusses how we could explain the everyday experience of the absence of biological antimatter in our world by maintaining the validity of relativistic quantum mechanics but adopting the currently controversial idea that an electron like a positron could interact electrically with itself.

This is not an easy-to-read book because it assumes the validity of relativistic quantum mechanics [2, 4], which is not an easy topic. Nevertheless, the author uses a simpler introduction to relativistic quantum mechanics recently proposed [4]. There is no use of the Dirac wave equation in this approach to relativistic quantum mechanics. Instead of the Dirac equation [2], this book is based on a simpler Schrödinger-like but relativistic wave equation, the so-called Grave de Peralta equation [4]. For simplicity, the discussion is based on the solution of the Grave de Peralta equation for the infinity well. This is the simplest model for a spatially localized relativistic quantum particle with mass. A semiquantitative discussion of the consequences of adding the interaction of the quantum particle with itself is presented.

Finally, the matter-antimatter symmetry is broken by postulating that a particle electrically interacts with itself in a different way than the corresponding antiparticle interacts with itself. It is shown a notable consequence of this hypothesis: this theory can explain our everyday experience of living in a world where only biological beings made of matter exist. This strongly suggests that current relativistic quantum mechanics should be expanded by including the interaction of each quantum particle and antiparticle with itself.

It is worth noting that this monograph only refers to gravitational and electrical interactions. No reference to other kinds of interactions is needed to explain the formation of inorganic and organic molecules, viruses, cells, and other living forms.

The author invites the readers to follow him through this exercise of scientific curiosity.

Chapter 1

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The Simplest Model of a Spatially Confined Quantum Particle

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The description of classical particles corresponds with our everyday experiences of objects, small and humongous, which we perceive as spatially localized. We can certainly talk about the size of a classical particle. Moreover, our everyday experiences tell us that we are surrounded by classical particles. In Classical Mechanics, the translation of a classical particle is described through the description of the translation of its center of mass [5]. The center of mass is a mathematical point of null size. However, the classical particle has no null size.

Quantum particles are different. There is a wave or quantum field associated with each quantum particle [6]. Nevertheless, like every particle, quantum particles are supposed to be spatially localized in some way. A quantum particle could be found anywhere its wavefunction or quantum field exists [6]; therefore, the particle character of a quantum particle could be associated with the spatial extension or size of the wavefunction associated with it. For instance, the Hydrogen atom is very small, but it is not a mathematical point without size. This is because the electron in the Hydrogen atom could be anywhere the wavefunction associated with the electron exists. The electron’s wavefunction is spatially localized around the proton in the Hydrogen atom. This determines the small size of the Hydrogen atom. In Quantum Mechanics, a wave packet is usually used as the wave function associated with a free particle [6]. The spatial localization of the wave packet in a small region corresponds to the intuitive spatial localization that every particle should have.

In nonrelativistic quantum mechanics, the wave function associated with a quantum particle is found by solving the Schrödinger equation [6]. For this reason, some mathematical skills are required for a full understanding of Quantum Mechanics. Nevertheless, to reach the maximum audience possible, we will use in this monograph the minimum amount of mathematics needed to achieve a good understanding of the central topic addressed by this monograph, which is answering the question “Where the biological antimatter is?”

SECTION 1A. THE SCHRÖDINGER EQUATION

The basic difference between a classical and a quantum particle is that there is a wave associated with a quantum particle [4]. There is no wave associated with a classical particle. Different interpretations of quantum mechanics give different answers to the nature of the wave associated with a quantum particle [4,6]. Nevertheless, all interpretations of quantum mechanics coincide in that the wavefunction corresponding to the wave associated with a nonrelativistic quantum particle is a solution of the Schrödinger equation [4,6]. The one-dimensional Schrödinger equation for a nonrelativistic quantum particle with mass *m* is given by the following expression [4,6]:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)\,. \tag{1.1}$$

The solution of this wave equation is the wavefunction ψ that depends on the spatial (*x*) and temporal (*t*) variables. For simplicity, we discuss here only the one-dimensional Schrödinger equation. However, space is tridimensional; therefore, a more realistic wave function will depend on three spatial variables [4,6]. In Eq. (1.1), $V(x)$ is a potential that only depends on the spatial variable, $\hbar = h/2\pi$ is the reduced Plank constant, and $i = \sqrt{-1}$ is the imaginary unit. Note that *m* and *V* are the only properties of the particle and the medium where the particle is, respectively, that are explicitly included in the Schrödinger equation. The wave associated with a free nonrelativistic quantum particle is a solution of the simplest Schrödinger equation possible ($V = 0$):

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\,\psi(x,t). \tag{1.2}$$

Looking for stationary solutions of the form:

$$\psi(x,t) = \varphi(x)T(t)\,. \tag{1.3}$$

Substituting Eq. (1.3) in Eq. (1.2) and dividing the result by φT, we obtain:

$$i\hbar\frac{\frac{d}{dt}T}{T} = -\frac{\hbar^2}{2m}\frac{\frac{d^2}{dx^2}\varphi}{\varphi}. \tag{1.4}$$

Both sides of Eq. (1.4) should be equal for all values of *x* and *t*. Consequently, each side of Eq. (1.4) should be equal to the same constant. Let's call it for convenience *E*. We will see later that *E* is an excellent name because, as we will find out later, *E* is the eigenvalue of the eigenequation corresponding to the total energy of the particle. In this way we can reduce the task of solving the Schrödinger equation, that is an equation in partial derivatives involving second-order derivatives, to the simpler task of solving a system of the following two differential equations:

$$i\hbar\frac{d}{dt}T = ET\,. \tag{1.5}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi = E\varphi\,. \tag{1.6}$$

For obvious reasons, Equation (1.6) is called the time-independent Schrödinger equation [6]. By solving the time-independent Schrödinger equation, we can find the eigenfunctions (φ) and the eigenvalues (E) corresponding to the energy of the free nonrelativistic quantum particle. It is easy to check by simple substitution on it, that the following function is a solution of Eq. (1.5):

$$T(t) = e^{-\frac{i}{\hbar}Et}\,. \qquad (1.7)$$

Consequently, if φ is a solution of the time-independent Schrödinger equation (Eq. (1.6)), then the following is a stationary solution of the Schrödinger equation (Eq. (1.2)):

$$\psi(x,t) = \varphi(x)e^{-\frac{i}{\hbar}Et}\,. \qquad (1.8)$$

The simplest way to obtain a reasonably good approximation to the wavefunction (φ) of a free quantum particle is assuming that, for some reason, the wavefunction associated with the particle is absolutely confined in a very small space region [4].

SECTION 1B. INFINITE ONE-DIMENSIONAL WELL

While the temporal dependence of the stationary solutions of the Schrödinger equation is always given by Eq. (1.7), the eigenfunctions (φ) and eigenvalues (E) of the Schrödinger equation (Eq. (1.1)) are different for different potentials [4,6]. Possibly, the crudest but simplest possible model of a "free" quantum particle is then a particle absolutely confined in the open segment $0 < x < L$. The wave function in such a model should be null in the rest of the line. In particular, ψ(0, t) = ψ(L, t) = 0 at all times. This supposes that the particle should be moving in a repulsive potential infinitely large in the regions $0 \leq x$ and $x \geq L$. The simplest of such potentials is the so-called infinite one-dimensional well [4,6]:

$$V(x) = \begin{matrix} 0, \; if \; 0 < x < L \\ +\infty, \; otherwise \end{matrix}. \qquad (1.9)$$

This choice of potential implies that the particle is like a free particle inside of the well. This problem was reduced to solving the following mathematical problem:

$$\begin{matrix} i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\,\psi(x,t), & if \; 0 < x < L \\ \psi(x,t) \equiv 0 & , \quad otherwise \end{matrix}\,. \qquad (1.10)$$

The stationary solutions of Eq. (1.10) are of the form given by Eq. (1.8), where φ is a solution of:

$$\begin{matrix} \frac{\partial^2}{\partial x^2}\,\varphi(x,t) = -k^2\varphi, \quad if \; 0 < x < L \\ \varphi(x) \equiv 0, \quad \text{otherwise} \end{matrix}\,, \text{ with } k = \frac{\sqrt{2mE}}{\hbar}. \qquad (1.11)$$

If $E \geq 0$, then $k \geq 0$; therefore, the general solutions of the time-independent Schrödinger equation in Eq. (1.11) are:

$$\varphi(x) = Asin(kx) + Bcos(kx). \tag{1.12}$$

But:

$$\varphi(0) = B = 0. \tag{1.13}$$

Thus:

$$\varphi(L) = Asin(kL) \Rightarrow k_n = \frac{n\pi}{L}, \text{ with } n = 1, 2, ... \ . \tag{1.14}$$

Therefore, the eigenfunctions and eigenvalues of the time-independent Schrödinger equation in Eq. (1.11) are:

$$\varphi_n(x) = Asin\left(\frac{n\pi}{L}x\right), \text{ and } E_n = \frac{\hbar^2 k_n^2}{2mL^2} = \frac{\hbar^2\pi^2}{2mL^2}n^2 \text{ , with } n = 1, 2, ... \tag{1.15}$$

The constant *A* can be found from the normalization condition [4,6]:

$$\langle\varphi_n|\varphi_n\rangle = 1 = \int_0^L A^2 sin^2\left(\frac{n\pi}{L}x\right)dx = A^2 L \ \Rightarrow A = \frac{1}{\sqrt{L}}. \tag{1.16}$$

Consequently, the stationary solutions of the Schrödinger equation in a one-dimensional infinite well are:

$$\psi_n(x,t) = \frac{1}{\sqrt{L}} sin(k_n x) e^{-\frac{i}{\hbar}E_n t}. \tag{1.17}$$

We should stop now and discuss the physical meaning of the mathematical results that we obtained. We should not get lost in mathematical formalism and forget why we were interested in solving Eq. (1.10). We proposed Eq. (1.10) as the Schrödinger equation problem corresponding to a simple but crude approximation of a quantum particle completely confined in a small space region. This is a simple approximation because it is a one-dimensional problem and because the infinite well potential is used. It is a crude approach for the same reasons. Nevertheless, as will be discussed below, the obtained results can explain why Hydrogen atoms are stable and why their spectra are formed by a discrete set of bright or dark lines [4].

The only possible values of the energy of the stationary states of a quantum particle with mass *m* that is confined in a small spatial region are given by Eq. (1.15). Therefore, the minimum possible energy value is:

$$E_1 = \frac{\hbar^2\pi^2}{2mL^2} > 0. \tag{1.18}$$

The existence of a non-null minimum value of the energy means that the spatially localized particle cannot lose more energy if it is in its ground state (*n* = 1). This explains the stability of the Hydrogen atom. Moreover, Eq. (1.15) follows that a quantum particle with mass *m* that is confined in a small spatial region must have a discrete frequency spectrum. It should be noted that Eq. (1.15) does not correctly predict the experimentally observed spectrum of the Hydrogen atom. This could be expected because the electron in the Hydrogen atom is not moving in an infinite well potential (Eq. (1.9)) but in the Coulomb potential produced by the proton. Nevertheless, it is amazing that both the stability of the Hydrogen atoms and the discrete character of their spectra are just consequences of the spatial localization of the quantum particle.

It should also be noted that the energy values given by Eq. (1.15) correspond to the internal kinetic energy of the quantum particle because $V = 0$ inside the well. For instance, the electron in the Hydrogen atom has a minimum internal kinetic energy. In addition, a free Hydrogen atom can realize a translation movement with any possible kinetic energy value from a continuum set of positive values.

SECTION 1C. BOHR RADIUS

The size of the Hydrogen atom can be semi-quantitatively obtained by realizing that, in the Hydrogen atom, the electron is approximately trapped in a localized spherical region of radius r. Therefore, the Bohr radius (r_B) can easily be obtained as the value of r that minimizes the sum of the particle-in-a-box kinetic energy (Eq. (1.18) with $L = r$), plus the potential energy of the slow-moving electron in the Hydrogen atom [4,10]:

$$E_{Sch}(r) \approx \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0 r}. \qquad (1.19)$$

The first term of $E_{Sch}(r)$, corresponds to the non-relativistic kinetic energy of the ground state of a trapped and slow-moving quantum particle with the electron mass ($m = m_e$). The second term corresponds to the potential energy associated with the Coulombic attraction between a particle, with a charge equal to the electron charge ($-e$), and a positive charge $+e$ placed at $r = 0$ [5]. $E_{Sch}(r)$ has a minimum when [4, 10]:

$$r = r_B = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{1}{\alpha}\bar{\lambda}_C, \text{ with } \bar{\lambda}_C = \frac{\hbar}{m_e c} \text{ and } \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \qquad (1.20)$$

In Eq. (1.20), c is the speed of the light in the vacuum and ε_0 is the dielectric constant of the vacuum. Therefore, the size of the Hydrogen atom is approximately $1/\alpha \approx 137$ times the electron's reduced Compton wavelength, which confirms the initial slow-moving assumption. The Bohr radius is small ($r_B \approx 0.05$ nm) but is not null.

Chapter 2

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The Simplest Model of a Spatially Confined Relativistic Quantum Particle

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The kinetic energy of classical particles is always positive [5]. As discussed in Chapter 1, the kinetic energy of nonrelativistic quantum particles with mass (*m*) is always positive too [6]. However, relativistic quantum particles can exist in two kinds of quantum states [2,4]. Common quantum states are states like nonrelativistic quantum states. In addition, relativistic quantum particles can exist in exotic states which are very different than nonrelativistic quantum states. In contrast with classical particles and nonrelativistic quantum particles, if a free relativistic quantum particle with mass is in an exotic state, then its kinetic energy has a negative value [4].

The quantum states that are solutions of relativistic (Lorentz's covariant) wave equations can be grouped into two branches [2,4]. In the first branch, referred from this point on as the common branch, the total relativistic energy of the quantum particle is $E_T = E + mc^2$. In the other branch, referred to from this point on, as the exotic branch, the total energy of the quantum particle is $E_T = E' - mc^2$. The apostrophe (') attached to the variable representing a magnitude will be used to exemplify the magnitude in an exotic quantum state.

In relativistic quantum mechanics, there is an antiparticle associated with each elemental particle. The associated particle and the antiparticle have the same mass and charges of equal magnitude. On the contrary, the associated particle and antiparticle have charges with opposite signs. For instance, a positron is the antiparticle associated with the electron. As will be discussed below, there is a close relationship between the antiparticle common states with $E_{Ta} = E_a + mc^2$ and the particle exotic states with $E_T = E' - mc^2$ [2,4].

SECTION 2A. SPECIAL RELATIVITY THEORY

The special theory of relativity was developed in 1905 by Albert Einstein considering only the existence of classical particles [7]. In special relativity theory, free particles have constant mass $m > 0$ and kinetic energy $K \geq 0$. The relationship between the total energy (E_T) and the linear momentum of a relativistic free particle is given by the following equation [4,7]:

$$E_T^2 = p^2c^2 + m^2c^4, \text{ with } p^2 = p_x^2 + p_y^2 + p_z^2 \,. \tag{2.1}$$

Or:

$$E_T = \pm\sqrt{p^2c^2 + m^2c^4}. \tag{2.2}$$

For classical particles the + sign in Eq. (2.2) must be taken; therefore:

$$E_T = \sqrt{p^2c^2 + m^2c^4}. \tag{2.3}$$

From Eq. (2.3), it follows when $p = 0$, then:

$$E_T = E_m = mc^2. \tag{2.4}$$

There is then an energy ($E_m = mc^2$) associated with the mass of a relativistic particle. Consequently:

$$E_T = E + mc^2, \text{ with } E = K. \tag{2.5}$$

Substituting E_T with $K + mc^2$ in Eq. (2.3), we obtain:

$$K + mc^2 = \sqrt{p^2c^2 + m^2c^4} = \gamma mc^2, \text{ with } \gamma = \sqrt{1 + \frac{p^2}{m^2c^2}} \geq 1. \tag{2.6}$$

Note that Eq. (2.6) gives an alternative formula for the Lorentz factor. This is because substituting in Eq. (2.6) p by $\gamma m\text{v}$, which is the equation corresponding to the relativistic linear momentum of a relativistic particle with mass m and speed v, we obtain the customary definition of the Lorentz factor [7]:

$$\gamma = \sqrt{1 + \gamma^2\frac{\text{v}^2}{c^2}} \Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{\text{v}^2}{c^2}}} . \tag{2.7}$$

From Eq. (2.6), we obtain the kinetic energy of a relativistic free particle is given by:

$$K = (\gamma - 1)mc^2 \geq 0. \tag{2.8}$$

Substituting Eq. (2.8) in Eq. (2.5), we obtain the following equations of:

$$E_T = \gamma mc^2. \tag{2.9}$$

Along with:

$$K = E = (\gamma - 1)mc^2. \tag{2.10}$$

From Eq. (2.10), we obtain another very useful alternative formula for γ for a relativistic free particle [4]:

$$\gamma = 1 + \frac{K}{mc^2}. \tag{2.11}$$

Note that $\gamma \geq 1$ in Eq. (2.11) because for a classical free particle $E = K \geq 0$. An unfamiliar but particularly useful alternative equation for K can be obtained from Eq. (2.6):

$$\gamma^2 = 1 + \frac{p^2}{m^2c^2}. \quad (2.12)$$

Therefore:

$$(\gamma + 1)(\gamma - 1)mc^2 = \frac{p^2}{m}. \quad (2.13)$$

Thus [4]:

$$(\gamma - 1)mc^2 = K = \frac{p^2}{(\gamma+1)m}. \quad (2.14)$$

Note that for classical particles γ ≈ 1 in the nonrelativistic limit; thus, this limit *K* and *p* are given by the nonrelativistic formulas [5]:

$$K = \frac{p^2}{2m}, \text{ with } p = m\text{v}. \quad (2.15)$$

As it will be shown later, it is useful to introduce the concept of the effective relativistic mass of a free particle as [4]:

$$\mu = \frac{1+\gamma}{2} m \geq m, \text{ with } \gamma = 1 + \frac{K}{mc^2}. \quad (2.16)$$

Thus:

$$\mu = \left(1 + \frac{K}{2mc^2}\right) m \geq m. \quad (2.17)$$

Using Eqs. (2.17) and (2.16), Eq. (2.14) can be rewritten as [4, 8-10]:

$$K = \frac{p^2}{2\mu} \geq 0. \quad (2.18)$$

In this monograph, Eq. (2.16) to (2.18) are the more important relativistic equations valid for free classical particles.

SECTION 2B. INTRODUCING SPECIAL RELATIVITY IN QUANTUM MECHANICS

In contrast to classical particles, a free relativistic quantum particle in exotic states can have $K' < 0$ [2, 4]. Nevertheless, free relativistic quantum particles can also exist in common quantum states where $E_T = E + mc^2$ and the particle has $K > 0$. For these common states, all the formulas discussed in Section 2.1 for classical particles are valid for a free relativistic quantum particle.

In addition to quantum states where the free relativistic quantum particle has $K > 0$, a free relativistic quantum particle can be in other exotic quantum states where $E_T = E' - mc^2$. In contrast with classical particles, free relativistic quantum particles in these exotic states have kinetic energy $K' < 0$. For these exotic states, the formulas discussed above should be modified. The special theory of relativity was developed by Albert Einstein for classical particles. Therefore, we must be careful in extrapolating the valid results for classical particles to relativistic quantum particles in exotic states where $E_T = E' - mc^2$ [4].

There exist particles and antiparticles in Mother Nature. An antiparticle is a particle that has the same mass as the associated particle but contains an opposite charge. For instance, the positron is the antiparticle of the electron. Although they have the same mass, the positron and electron electric charges are e and $-e$, respectively. According to Dirac's Hole Theory, the existence of a hole in Dirac's Sea means there exists an unoccupied exotic quantum state of a free quantum particle with total energy $E_T = E' - mc^2$ [2, 4]. This hole is perceived as an antiparticle moving with total energy $E_{Ta} = E_a + mc^2$ and $E_a = -E'$. Consequently, due to the relation of $E_a = -E'$, we can start by obtaining the relativistic equations that are valid for the free antiparticle in the common states with $E_{Ta} = E_a + mc^2$. After they are found and known, the relativistic equations valid for the exotic quantum states of the corresponding particle can be deducted from them [4].

An antiparticle is also a "particle" that is always experimentally observed with positive values of its kinetic energies ($K_a > 0$). Consequently, all the equations obtained in Section 2.1 for a free classical particle are also valid for a free antiparticle in a common quantum state. This means that Eq. (2.16) to (2.18) should be rewritten for an antiparticle in a common state in the following way [4]:

$$K_a = \frac{p^2}{2\mu_a}, \text{ with } \mu_a = \left(\frac{\gamma_a+1}{2}\right)m = \left[1 + \frac{K_a}{2mc^2}\right]m. \tag{2.19}$$

For a free antiparticle with $E_{Ta} = E_a + mc^2$, Eqs. (2.1) is also valid. Therefore:

$$E_{Ta}^2 = p^2c^2 + m^2c^4, \quad \text{with} \quad p^2 = p_x^2 + p_y^2 + p_z^2\,. \tag{2.20}$$

Or:

$$E_{Ta} = E_a + mc^2 = \sqrt{p^2c^2 + m^2c^4}. \tag{2.21}$$

Using the relation $E' = -E_a$, we obtain:

$$-E_{Ta} = -E_a - mc^2 = E' - mc^2 = E_T = -\sqrt{p^2c^2 + m^2c^4}. \tag{2.22}$$

Therefore, if the free particle is in an exotic state with $E_T = E' - mc^2$, from Eqs. (2.21) and (2.22) follow when $p = 0$ that:

$$E_T = -E_{Ta} = -E_m = -mc^2. \tag{2.23}$$

Therefore, mc^2 is the absolute minimum value of E_T, if a free relativistic quantum particle is in a common state where $E_T = E + mc^2$ (Eq. (2.4)). However, $-mc^2$ is the absolute maximum value of E_T, if a free relativistic quantum particle is in an exotic state where $E_T = E' - mc^2$ [4].

As expected, from equation (2.22) follows that a free antiparticle in a common state satisfices Eq. (2.5):

$$E_{Ta} = E_a + mc^2, \text{ with } E_a = K_a. \tag{2.24}$$

Thus, in the exotic states the following equation is valid:

$$E_T = -E_{Ta} = -(E_a + mc^2) = -K_a - mc^2. \tag{2.25}$$

We can rewrite Eq. (2.25) as:

$$E_T = E' - mc^2, \text{ with } E' = K' = -E_a, \text{ and } K' = -K_a. \tag{2.26}$$

Therefore, a free relativistic quantum particle has a value of $E = K > 0$ in a common state where $E_T = E + mc^2$ (Eq. (2.5)). However, $E' = K' = -K_a < 0$ in an exotic state where $E_T = E' - mc^2$.

Also, as expected for a free antiparticle in a common site, by substituting E_{Ta} by $K_a + mc^2$ in (2.21) we obtain the same equation that is valid for the corresponding free particle in a common state (Eq. (2.6)):

$$K_a + mc^2 = \sqrt{p^2c^2 + m^2c^4} = \gamma_a mc^2, \; with \; \gamma_a = \sqrt{1 + \frac{p_a^2}{m^2c^2}} \geq 1. \tag{2.27}$$

From Eqs. (2.26) and (2.27), we obtain [4]:

$$K_a = (\gamma_a - 1)mc^2 = -K'. \tag{2.28}$$

Substituting K_a given by Eq. (2.27) in Eq. (2.25), we obtain the following equations:

$$E_{Ta} = \gamma_a mc^2 . \tag{2.29}$$

And:

$$E_a = K_a = (\gamma_a - 1)mc^2. \tag{2.30}$$

As expected, Eqs. (2.29) and (2.30) for a free antiparticle in a common state match Eqs. (2.9) and (2.8), respectively, for a free relativistic quantum particle in a common state. Also, from Eq. (2.30), we obtain an alternative formula for γ_a:

$$\gamma_a = 1 + \frac{K_a}{mc^2}. \tag{2.31}$$

Note that $\gamma_a > 1$ in Eq. (2.31) when $K_a > 0$. Also, for free particles and antiparticles in common states $\gamma_a = \gamma$, with γ given by Eq. (2.11).

We are interested in obtaining equations valid for a free relativistic quantum particle in an exotic state. We want to obtain these equations from the equations valid for the corresponding free antiparticle in a common state. If we substitute K' given by Eq. (2.28) in Eq. (2.26), we obtain the following equations for a free relativistic quantum particle in an exotic state [4]:

$$E_T = -\gamma_a mc^2 = \gamma' mc^2. \tag{2.32}$$

And:

$$E' = K' = (\gamma' + 1)mc^2. \tag{2.33}$$

From Eq. (2.33), we obtain a formula for γ':

$$\gamma' = -1 + \frac{K'}{mc^2} \tag{2.34}$$

Note that in Equation (2.32) we defined $\gamma' = -\gamma_a$ as the Lorentz factor corresponding to a free relativistic quantum particle in an exotic state where $E_T = E' - mc^2$. In contrast with the Lorentz factor for free classical particles, $\gamma' < -1$ because $\gamma_a > 1$.

Also, due to $\gamma' = -\gamma_a$, Eq. (2.12) is also valid for γ', thus:

$$(\gamma' - 1)(\gamma' + 1)mc^2 = \frac{p^2}{m}. \tag{2.35}$$

From Eqs. (2.33) and (2.35), we obtain [4]:

$$K' = \frac{p^2}{(\gamma'-1)m} = \frac{p^2}{2\mu'}, \text{ with } \mu' = \left(\frac{\gamma'-1}{2}\right)m = \left(-1 + \frac{K'}{mc^2}\right)m. \tag{2.36}$$

We have then obtained similar kinetic energy equations, $K = p^2/2\mu$ (Eq. (2.18)) and $K' = p^2/2\mu'$. These equations are valid for both kinds of quantum states. However, the effective relativistic masses μ (Eq. (2.17)) and μ' are different. The equations relating μ and γ (Eq. (2.16)) are also different than the equations relating μ' and γ' (Eq. (2.36)). Also, due to the relation of $E_a = -E'$, it follows that for a free quantum particle $-\mu' = \mu_a$ (Eqs. (2.36) and (2.19)).

In summary, if a free relativistic quantum particle is in a common state, then Eq. (2.16) to (2.18) are the more important relativistic equations. However, if a free relativistic quantum particle is in a exotic state, then these equations should be substituted by Eqs. (2.34) and (2.36). This is also valid for free antiparticles.

It should be noted that in Sections 2.a and 2.b, the focus has been put on free quantum particles. This is because this monograph is based on the simplest model of a localized quantum particle, the infinite well [4]. The quantum particle is free inside of the infinite well because $V = 0$ there.

SECTION 2C. GRAVE DE PERALTA EQUATIONS

The mass is the only feature of a free quantum particle present in the Schrödinger equation (Eq. (1.2)). This suggests the unconventional idea that it is possible to obtain two Schrödinger-like but relativistic equations just by substituting the mass of the particle (*m*) in the Schrödinger equation by the effective relativistic masses of the quantum particle in the common (μ) and exotic (μ') states. This formal substitution results in the so-called Grave de Peralta equations for a free relativistic quantum particle [4, 8-10]:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\Psi, \text{ with } \mu = \left(1+\frac{K}{2mc^2}\right)m. \tag{2.37}$$

And

$$i\hbar\frac{\partial}{\partial t}\Omega = -\frac{\hbar^2}{2\mu'}\nabla^2\Omega, \text{ with } \mu' = \left(-1+\frac{K'}{2mc^2}\right)m. \tag{2.38}$$

If a free relativistic particle is in a common state with energy $E_T = E + mc^2$, then Eq. (2.37) is valid. If a free relativistic particle is in an exotic state with energy $E_T = E' - mc^2$, then Eq. (2.38) is valid. A general discussion about the Grave de Peralta equations could be found elsewhere [4]. Here, it is shown below an intuitive but precise way for obtaining these relativistic equations, which is based on using a first quantization procedure [2, 4, 6, 8-10].

For instance, the Schrödinger equation for a free quantum particle can be obtained from the following nonrelativistic classical mechanic equation (Eq. (2.15)):

$$E_T = K = \frac{p^2}{2m}, \text{ with } p = m\text{v}. \tag{2.39}$$

The first quantization procedure consists of changing the total energy of the particle (E_T) and its linear momentum (*p*) in Eq. (2.39) by the corresponding quantum operators [4, 6]:

$$E_T \to \hat{H} = i\hbar\frac{\partial}{\partial t}, \text{ and } K \to \hat{K} = \frac{\hat{p}^2}{2m} \text{ with } \hat{p} = -i\hbar\frac{\partial}{\partial x}. \tag{2.40}$$

This procedure allows for formally obtaining Eq. (1.2). Similarly, the Grave de Peralta equations can be obtained using Eqs. (2.18) and (2.36). From Eqs. (2.5) and (2.18) follow the following equation for a free relativistic quantum particle in a common state:

$$E_T = K + mc^2, \text{ with } K = \frac{1}{2\mu}p^2, \text{ and } \mu = \left(1+\frac{K}{2mc^2}\right)m. \tag{2.41}$$

Note that $K > 0$ and $\mu > m$. Making in Eq. (2.41) the formal first quantization substitutions given by Eq. (2.40), we obtain [4, 8-10]:

$$\hat{H}\phi = \left(\hat{K} + mc^2\right)\phi \;\Leftrightarrow i\hbar\frac{\partial}{\partial t}\phi = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\phi + mc^2\phi. \tag{2.42}$$

Equation (2.42) resembles the one-dimensional Schrödinger equation for a free quantum particle. This similarity can be improved by introducing a new wavefunction:

$$\Psi = \phi e^{i\frac{mc^2}{\hbar}t}. \tag{2.43}$$

Note that both wavefunctions represent the same probability density $\rho = |\Psi|^2 = |\phi|^2$. Finally, Eq. (2.37) can be obtained by substituting ϕ given by Eq. (2.43) in Eq. (2.42).

Alternatively, from Eqs. (2.26) and (2.36) follows the following equation for a free relativistic quantum particle in an exotic state [4]:

$$E_T = K' - mc^2, \text{ with } K' = \frac{1}{2\mu'}p^2, \text{ and } \mu' = \left(-1 + \frac{K'}{2mc^2}\right)m\,. \tag{2.44}$$

Note that $K' < 0$ but $\mu' < -1$. As discussed in Sections 2.a and 2.b, Eq. (2.44) is not a proper equation in special relativity theory because this theory was developed for classical particles with $m > 0$ and $K \geq 0$. Nevertheless, we can obtain Eq. (2.38) by making in Eq. (2.44) the formal first quantization substitutions given by Eq. (2.40):

$$\hat{H}\Sigma = \left(\widehat{K'} - mc^2\right)\Sigma \;\Leftrightarrow i\hbar\frac{\partial}{\partial t}\Sigma = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\Sigma - mc^2\Sigma. \tag{2.45}$$

Equation (2.45) also resembles the one-dimensional Schrödinger equation for a free quantum particle. This similarity can be improved by introducing a new wavefunction:

$$\Sigma = \Omega e^{-i\frac{mc^2}{\hbar}t}. \tag{2.46}$$

Note that both wavefunctions represent the same probability density $\rho = |\Sigma|^2 = |\Omega|^2$. Finally, Eq. (2.38) can be obtained by substituting Σ given by Eq. (2.46) in Eq. (2.45).

The pair of Grave de Peralta equations is equivalent to the well-known Klein-Gordon equation for a free spinless quantum particle with mass [2, 4]. This means that the solutions of the Klein-Gordon equation with $E_T = K + mc^2$ can be found by solving Eq. (2.37). Also, the solutions of the Klein-Gordon equation with $E_T = K' - mc^2$ can be found by solving Eq. (2.38). This can be easily demonstrated [4].

The time-independent equation corresponding to Eq. (2.37) is:

$$-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\varphi = E\varphi, \text{ with } \mu = \left(1 + \frac{E}{2mc^2}\right)m > m, \text{ and } E = K. \tag{2.47}$$

By pre-multiplying both sides of the Equation (2.47) by μ/m, substituting μ with his value in Equation (2.47) into the resulting equation, and after some algebraic manipulation, we obtain [4]:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi = \left(E + \frac{E^2}{2mc^2}\right)\varphi. \tag{2.48}$$

We can also arrive at Eq. (2.48) but from starting from the one-dimensional Klein-Gordon equation for a free spinless particle with mass [2, 4]:

$$\left(i\hbar\frac{\partial}{\partial t}\right)^2 \psi_{KG} = -\hbar^2 c^2 \frac{\partial^2}{\partial x^2}\psi_{KG} + m^2c^4\psi_{KG}. \tag{2.49}$$

We can obtain the time-independent Klein-Gordon equation corresponding to Eq. (2.49) by looking for stationary solutions of the form:

$$\psi_{KG}(r,t) = \varphi_{KG}(r)e^{-\frac{i}{\hbar}E_T t}\ . \tag{2.50}$$

Substituting Eq. (2.50) in Eq. (2.49), and after several algebraic manipulations, we obtain the time-independent one-dimensional Klein-Gordon equation [2, 4]:

$$-\hbar^2c^2\frac{\partial^2}{\partial x^2}\varphi_{KG} = (E_T^2 - m^2c^4)\varphi_{KG}\ . \tag{2.51}$$

Then, substituting E_T by E + mc^2, and after some algebraic manipulations, we obtain [4]:

$$-\hbar^2c^2\frac{\partial^2}{\partial x^2}\varphi_{KG} = 2mc^2\left(E + \frac{E^2}{2mc^2}\right)\varphi_{KG}. \tag{2.52}$$

Then, after dividing by 2mc^{2} on both sides of Eq. (2.52), we obtain Eq. (2.48). We have then demonstrated that by solving either Eq. (2.37) or Eq. (2.49), we could obtain the energies (E = K) and wavefunctions (φ) corresponding to a free spinless relativistic quantum particle with mass containing a total energy E_T = E + mc^2. Note that this demonstration requires substituting E_T by E + mc^2 in Eq. (2.51).

In a similar way, the time-independent equation corresponding to Eq. (2.38) is:

$$-\frac{\hbar^2}{2\mu'}\frac{\partial^2}{\partial x^2}\vartheta = E'\vartheta,\ \text{ with } \mu' = \left(-1 + \frac{E'}{2mc^2}\right)m > m, \text{and } E' = K'. \tag{2.53}$$

By pre-multiplying both sides of the Equation (2.53) by μ' /m, substituting μ' with his value in Equation (2.53) into the resulting equation, and after some algebraic manipulation, we obtain [4]:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\vartheta = \left[E' - \frac{(E')^2}{2mc^2}\right]\vartheta. \tag{2.54}$$

We can also arrive at Eq. (2.54) but from starting from the time-independent one-dimensional Klein-Gordon equation for a free spinless particle with mass (Eq. 2.51). After substituting E_T by E' - mc^2, and after some algebraic manipulations, we obtain [4]:

$$-\hbar^2 c^2 \frac{\partial^2}{\partial x^2} \varphi_{KG} = 2mc^2 \left[E' - \frac{(E')^2}{2mc^2}\right] \varphi_{KG}. \tag{2.55}$$

Then, after dividing by $2mc^{2}$ on both sides of Eq. (2.55), we obtain Eq. (2.54). We have then demonstrated that by solving either Eq. (2.38) or Eq. (2.49), we could obtain the energies ($E' = K'$) and wavefunctions (ϑ) corresponding to a free spinless relativistic quantum particle with mass containing a total energy $E_T = E'$ - mc^2. Note that this demonstration requires substituting E_T by E' - mc^2 in Eq. (2.54).

We need to solve just one Schrödinger equation for describing the quantum field associated to a nonrelativistic quantum particle, but we must solve two Schrödinger-like equations (Eqs. (2.37) and (2.38)) for a full description of the quantum field associated to a relativistic quantum particle. This fact can be summarized in the following colorful way:

There is no wave associated with a classical particle and a wave associated with a nonrelativistic quantum particle with mass. However, there are two waves associated with a relativistic quantum particle with mass. If the particle is in a common quantum state, its total energy is $E_T = E + mc^2$, but it is $E_T = E' - mc^2$ if the particle is in an exotic quantum state.

SECTION 2D. A RELATIVISTIC QUANTUM PARTICLE IN THE ONE-DIMENSIONAL INFINITE WELL

The kinetic energy of a nonrelativistic quantum particle trapped in an infinite well increases when the size of the well (L) decreases (Eq. (1.15)). Eventually, if the size of the well is too small, then $K \approx mc^2$ and the problem of a quantum particle confined in an infinite well transforms itself into a relativistic problem. Strictly, due to the possibility of the creation of particle-antiparticle pairs when $K > 2mc^2$, it is questionable that relativistic quantum particles could be confined in a very small region of space [4]. Nevertheless, for simplicity in a first approximation to a relativistic extension of the discussions made in Section 1.b, we could disregard the effects related to the creation of particle-antiparticle pairs.

If a free relativistic quantum particle could be absolutely confined in a small spatial region, then the wavefunctions of the free relativistic quantum particle in the common and exotic quantum states could be found solving problems very similar to Eq. (1.10) [4]:

$$\begin{array}{ll} i\hbar \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial x^2} \psi(x,t), & if\ 0 < x < L \\ \psi(x,t) \equiv 0 & ,\quad otherwise \end{array} . \tag{2.56}$$

And

$$\begin{matrix} i\hbar\frac{\partial}{\partial t}\Omega(x,t) = -\frac{\hbar^2}{2\mu'}\frac{\partial^2}{\partial x^2}\,\Omega(x,t), & if\ 0 < x < L \\ \psi(x,t) \equiv 0 & ,\quad otherwise \end{matrix} . \tag{2.57}$$

If a free relativistic particle is in a common state with energy $E_T = E + mc^2$, then we should solve Eq. (2.56). If a free relativistic particle is in an exotic state with energy $E_T = E' - mc^2$, then we should solve Eq. (2.57). Conveniently, Eqs. (2.56) and (2.57) can be obtained from Eq. (1.10) after substituting m by μ and μ', respectively. This suggests that we can solve Eq. (2.56) as Eq. (1.10) was solved in Section 1.b. Moreover, for finding the possible values of K and K', we only need to find the values of K because $K' = -K$.

We are now prepared to find out what happens to a relativistic quantum particle when it is trapped in a small spatial region, and the particle is in a common quantum state. Solving Eq. (2.56) provides a crude, but simple mathematical model for this riveting physical situation. As discussed above in Section 1.b, the spatial part and the energies of the corresponding stationary states are solutions of the following mathematical problem [4, 8-10]:

$$\begin{matrix} \frac{\partial^2}{\partial x^2}\varphi = -k^2\varphi, & if\ 0 \le x\ \le L \\ \varphi(x) \equiv 0, & \text{otherwise} \end{matrix} ,\ \text{with}\ k = \frac{\sqrt{2\mu E}}{\hbar}. \tag{2.58}$$

Note that the relativistic quantum particle is free inside the well; therefore, $E = K$ and μ given by Eq. (2.37) are constant inside of the well. In the non-relativistic limit, $E << mc^2$, so $\mu \approx m$. Therefore, in the non-relativistic limit, Eq. (2.58) reduces to the Eq. (1.11) that was solved in the previous Section 1.a. From Eqs. (2.58) and (2.37) follow that inside of the infinite well, $\mu \geq m$ and $k > 0$ are constants for a given value of $E > 0$. For that reason, the stationary solutions of Eq. (2.58) are given by the following expression [4, 8-10]:

$$\Psi_n(x,t) = \sqrt{\frac{2}{L}}sin(k_n x)e^{-\frac{i}{\hbar}E_n^{(rel)}t},\ \text{with}\ k_n = \frac{n\pi}{L},\ \text{and}\ \ n = 1,2,... \tag{2.59}$$

For the relativistic quantum particle, the energies depend on k_n and μ as E_n in Eq. (1.15) depends on k_n and m:

$$E_n^{(rel)} = \frac{\hbar^2 k_n^2}{2\mu L^2} = \frac{\hbar^2\pi^2}{2\mu L^2}n^2. \tag{2.60}$$

Equations (2.60) and (2.37) form the following system of two equations with two variables:

$$E_n^{(rel)} = \frac{\hbar^2\pi^2 n^2}{2\mu_n L^2},\ \mu_n = \left(1 + \frac{E_n^{(rel)}}{2mc^2}\right)m\,. \tag{2.61}$$

Therefore:

$$\frac{\hbar^2\pi^2 n^2}{2\mu_n L^2} = 2\left(\frac{\mu_n}{m} - 1\right)mc^2 \iff \mu_n^2 - m\mu_n - \frac{\hbar^2\pi^2 n^2}{4c^2L^2} = 0. \quad (2.62)$$

Due to $E = K > 0$, we should require that $\mu_n \geq m$. With that being said, the solutions of Eq. (2.62) are [4, 8-10]:

$$\mu_n = \frac{1}{2}\left(1 + \sqrt{1 + \frac{\hbar^2\pi^2 n^2}{m^2c^2L^2}}\right)m = \frac{1}{2}\left(1 + \sqrt{1 + \frac{n^2}{4}\left(\frac{\lambda_C}{L}\right)^2}\right)m\,, \text{ with } \lambda_C = \frac{h}{mc}. \quad (2.63)$$

Therefore:

$$\gamma_n = \frac{2\mu_n}{m} - 1 = \sqrt{1 + \frac{n^2}{4}\left(\frac{\lambda_C}{L}\right)^2} > 1. \quad (2.64)$$

In Eqs. (2.63) and (2.64), λ_C is the Compton wavelength associated with a particle of mass m. So, if the particle is in the ground state ($n = 1$), then the nonrelativistic limit occurs ($\gamma \approx 1$ and $\mu_n \approx m$) when $L >> \lambda_C$. The particle travels faster (γ increases) when the spatial confinement increases; that is when the width of the well (L) decreases. Substituting μ given by Eq. (2.63) in Eq. (2.61), we obtain [4, 8-10]:

$$E_n^{(rel)} = \frac{\hbar^2\pi^2 n^2}{(1+\gamma_n)mL^2} = \frac{\hbar^2\pi^2 n^2}{\left(1+\sqrt{1+\frac{n^2}{4}\left(\frac{\lambda_C}{L}\right)^2}\right)mL^2}\,. \quad (2.65)$$

In the non-relativistic limit (n small, $L >> \lambda_C$), Eq. (2.65) coincides with Eq. (1.15). If $L \approx \lambda_C/2$, then Eq. (2.65) reduces to:

$$E_n^{(rel)} = \frac{\hbar^2\pi^2 n^2}{\left(1+\sqrt{1+n^2}\right)mL^2}\,. \quad (2.66)$$

The minimum value of Eq. (2.66) is:

$$E_n^{(rel)} = \frac{\hbar^2\pi^2}{\left(1+\sqrt{2}\right)mL^2} > 0. \quad (2.67)$$

The ratio between the energies given by Eqs. (2.66) and (1.15) is:

$$\frac{E_n^{(rel)}}{E_n} = \frac{2}{\left(1+\sqrt{1+n^2}\right)}\,. \quad (2.68)$$

Consequently, when the particle moves faster (n increases), the energy of the highly confined particle decreases in comparison with the nonrelativistic energy value.

A more notable difference exists in the energy difference between consecutive energy levels ($\Delta E = E_{n+1} - E_n$) at the nonrelativistic and ultra-relativistic limits. At the nonrelativistic limit:

$$\Delta E_n = \frac{\hbar^2\pi^2}{2mL^2}\left[(n+1)^2 - n^2\right] = \frac{\hbar^2\pi^2}{2mL^2}(2n+1). \quad (2.69)$$

Therefore, Δ*E* increases as *n* increases at the nonrelativistic limit. However, using Eq. (2.67) for estimating Δ*E* at the ultra-relativistic limit (*n* >> 1), we obtain:

$$\Delta E_n = \frac{\hbar^2\pi^2}{mL^2}[(n+1)-n] = \frac{\hbar^2\pi^2}{mL^2}. \quad (2.70)$$

Therefore, Δ*E* is constant at the ultra-relativistic limit.

Finally, due to the relationship *K′* = -*K*, inside an infinite one-dimensional well, if the free relativistic particle with mass is in an exotic state, then its kinetic energies satisfice the following equation:

$$E_n^{(ex)} = -E_n^{(rel)} = -\frac{\hbar^2\pi^2n^2}{(1+\sqrt{1+n^2})mL^2}. \quad (2.71)$$

The maximum value of Eq. (2.71) is:

$$E_n^{(ex)} = -\frac{\hbar^2\pi^2}{(1+\sqrt{2})mL^2} < 0. \quad (2.72)$$

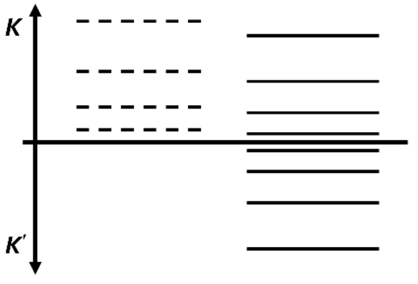

Fig. 2.1. Schematic of some kinetic energy values of a (discontinuous) non-relativistic and (continuous) relativistic quantum particle with mass in a one-dimensional infinite well

The schematic in Fig. 2.1 shows a comparison between some energy values of the stationary states of a (discontinuous) nonrelativistic and (continuous) relativistic quantum particle with mass *m* confined in a one-dimensional infinite well. Note that there are pairs of quantum states with total energies $E_T = K_n + mc^2$ and $E_T = -(K_n + mc^2)$ associated with a spatially confined relativistic quantum particle. Also, note that the non-relativist values of K_n (Eq. (1.15)) are larger than the corresponding values of K_n (same value of *n*) for the relativistic particle in a common quantum state (Eq. (2.66)).

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A Relativistic Quantum Particle May Interact with Itself

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There is no doubt about the existence of numerous quantum particles that interact with themselves. For instance, the Hydrogen atom exists because there is an electromagnetic interaction between the electron and the proton forming it. As discussed in Section 1.c, we could use a simple approach (Eq. (1.19)) to obtain satisfactory quantitative responses about some properties of quantum particles. Equation (1.19) is not a wave equation, but an approximate function given the total energy of the electron in the Hydrogen atom. The total energy is estimated as equal to the sum of the kinetic energy of a nonrelativistic quantum particle with mass trapped in a one-dimensional infinite well (Eq. (1.15)) plus the potential energy of the electron due to its Coulombic interaction with the proton. The Bohr radius was obtained by finding a local minimum of the function given by Eq. (1.19). The success of the obtained result justifies the use of this simple approach instead of trying to solve the corresponding wave equation, which may be a more formidable mathematical problem.

SECTION 3A. NUMBER OF ELEMENTS IN THE PERIODIC TABLE

A Hydrogen-like atom with atomic number *Z* is formed by a nucleus with positive charge +*Ze* and one electron. We could use the same approximated approach discussed in Section 1.c for obtaining a crude estimate of the size (radius) of a Hydrogen-like atom. Assuming that the electron is a nonrelativistic particle, this radius (r_Z) can easily be obtained as the value of *r* that minimizes the following function [4, 10]:

$$E_{Sch}(r) \approx \frac{\hbar^2}{2m_e r^2} - \frac{Ze^2}{4\pi\epsilon_0 r}. \qquad (3.1)$$

The first term of $E_{Sch}(r)$, corresponds to the nonrelativistic kinetic energy of the ground state of a trapped and slow-moving particle with the electron mass ($m = m_e$). The second term corresponds to the potential energy associated with the Coulombic attraction between a particle, with a charge equal to the electron charge (-*e*), and a positive charge +Z*e* placed at *r* = 0. The potential energy is negative due to the attractive interaction between the electron and the nucleus. $E_{Sch}(r)$ has a minimum when [4, 10]:

$$r = a = \frac{4\pi\epsilon_0\hbar^2}{m_e Z e^2} = \frac{r_B}{Z} = \frac{1}{\alpha Z}\lambda\!\!\!^{-}_C \text{, with } \lambda\!\!\!^{-}_C = \frac{\hbar}{m_e c} \text{ and } \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \qquad (3.2)$$

Therefore, Eq. (3.2) predicts that the size of a Hydrogen-like atom with atomic number Z should be approximately $1/\alpha Z$ times the electron's reduced Compton wavelength. For the Hydrogen atom Z = 1, thus $1/\alpha Z \approx 137$ times the electron's reduced Compton wavelength, which confirms the initial slow-moving assumption. However, the size of a Hydrogen-like atom with atomic number Z = 137 is approximately equal to the electron's reduced Compton wavelength. Therefore, the electron moves at relativistic speeds if $Z \approx 137$ because its kinetic energy approximately is:

$$K_{ƛ} \approx \frac{\hbar^2}{2m_e ƛ_C{}^2} = \frac{1}{2}\, m_e c^2. \tag{3.3}$$

This means that, if the atomic number $Z >> 1$, then the electron moves at relativistic speeds in the ground state of Hydrogen-like atoms. Therefore, to obtain a better estimate of the size of Hydrogen-like atoms, we should use Eq. (2.65) to modify Eq. (3.1) in the following way [4, 10]:

$$E_{GP}(r) \approx \frac{\hbar^2}{(\gamma+1)m_e r^2} - \frac{Ze^2}{4\pi\epsilon_0 r}\ \text{, with}\ \gamma = \sqrt{1 + \left(\frac{ƛ_C}{r}\right)^2} > 1. \tag{3.4}$$

$E_{GP}(r)$ has a minimum when [4, 10]:

$$r = r_Z = a\sqrt{1 - \left(\frac{ƛ_C}{a}\right)^2}\ \text{, with}\ a = \frac{r_B}{Z}. \tag{3.5}$$

If $Z << 1/\alpha \approx 137$, then $r_Z \approx a$, which is the value previously obtained for a slow-moving electron (Eq. (3.2)). However, when $Z >> 1$, the electron moves at relativistic speeds; this results in the square root factor in Eq. (3.5) becoming significant.

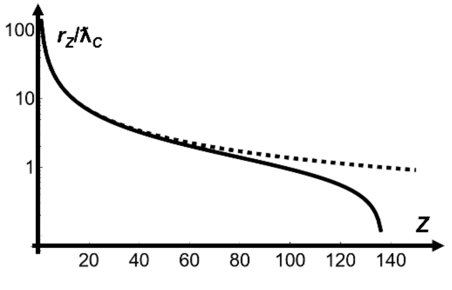

Fig. 3.1. (Discontinuous) Non-relativistic and (continuous) relativistic estimates of the radius (in reduced Compton wavelength units) of the common quantum field of the electron with $E_T = E + mc^2$ in Hydrogen-like atoms [4, 10]

As shown in Fig. 3.1, the relativistic correction to the size of the ground state of Hydrogen-like atoms becomes significant when $a \approx ƛ_C$. Moreover, the size of the

Hydrogen-like atom becomes undefined when $Z > 1/\alpha \approx 137$. This could be interpreted as a prediction about the impossibility of the natural existence of elements with $Z > 137$. This prediction matches the observed reality. No element with $Z > 118$ has ever been discovered.

As shown in Fig. 3.1, the common quantum field associated with the electron collapses to a point when $a \approx ƛ_C$. This suggests an interesting explanation for the finite number of elements in the Periodic Table of elements. Classical particles cannot form stable atoms. The observed stability of the atoms was one of the principal unexplained phenomena driving the development of Quantum Mechanics [4]. Classical particles cannot form stable atoms, but quantum particles can. It is the existence of a quantum field associated with a quantum particle that makes atoms stable. However, a relativistic electron moving in a Coulomb field lost the common wave associated with it when $a \approx ƛ_C$. This explains the finite number of elements in the Periodic Table. Note that nonrelativistic quantum mechanics would predict the existence of very heavy elements in Mother Nature. The finite number of elements in the Periodic Table can only be explained by combining quantum mechanics and special relativity.

It is worth noting that if $a \approx ƛ_C$, then the absolute value of the electron's potential energy can be estimated using Eq. (3.1):

$$|U_{ƛ}| \approx \frac{Ze^2}{4\pi\epsilon_0 ƛ} = m_e c^2. \tag{3.6}$$

Consequently, neither the kinetic energy of the electron (Eq. (3.3)) nor the absolute value of its potential energy reaches ultra-relativistic values larger than $2mc^2$. This justifies disregarding in our simple approach effects related with the creation of particle-antiparticle pairs.

Finally, it should be noted that there are two waves associated with a relativistic quantum particle. We have discussed above about the size of the quantum field corresponding to the common quantum states of a relativistic quantum particle moving in a Coulomb potential. If the relativistic quantum particle were in an exotic state, then Eq. (3.4) should be substituted by the following equation:

$$E'_{GP}(r) \approx -\frac{\hbar^2}{(\gamma+1)m_e r^2} - \frac{Ze^2}{4\pi\epsilon_0 r}. \tag{3.7}$$

However, Eq. (3.7) does not have a local extremum because both the kinetic energy of the quantum particle in exotic states and its potential energy are negatives. This means there is no collapse of the quantum field associated with the exotic states. We conclude then that a relativistic quantum particle cannot exist in a situation where the particle cannot have two waves but only one wave associated with it.

SECTION 3B. GRAVITY

How to combine quantum mechanics and general relativity theories is an open field of intense research. One of the simplest approaches in this direction consists in using the one-dimensional Schrödinger-Newton equation for a free quantum particle [11-12]:

$$i\hbar \frac{\partial}{\partial t}\psi(r,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}\,\psi(r,t) - \,Gm^2 \int \frac{|\psi(r',t)|^2}{|r'-r|}dr'. \qquad (3.8)$$

In Eq. (3.8), *G* is the gravitational constant [4, 5, 10]. Eq. (3.8) combines nonrelativistic quantum mechanics and classical gravity theories. The motivation for using the Schrödinger-Newton equation is that the only property of the particle included in the Schrödinger equation is the mass. This has been interpreted by some physicists as suggesting that there is a quantum field (wavefunction) associated with any object with mass. For instance, there should even be a wave function associated with the whole Universe. Our everyday experiences strongly suggest that any extended classical bodies of mass *m* should gravitationally interact with themselves. Black holes exist due to the gravitational interaction between the different parts of their original and spatially distributed mass. The enormous pressure existing inside planets acquires the same origin. If we extrapolate this to the quantum world, then any quantum particle with mass should interact gravitationally with itself. If the quantum particle moves slowly and the gravitation attraction is not too strong, then combining the Schrödinger equation with Newtonian gravitation is justified. Solving Eq. (3.8) is mathematically complicated because Eq. (3.8) is a nonlinear equation [11]; however, if we are interested in obtaining an estimate of the size of the quantum field associated with a free quantum particle with mass in a common state, we could simplify the mathematical problem to solve by proposing the following modification of Eq. (3.1) [4, 10]:

$$E_{Sch}(r) \approx \frac{\hbar^2}{2mr^2} - \,\frac{Gm^2}{r}. \qquad (3.9)$$

In Eq. (3.9), the Newtonian gravitational attraction of the particle with itself substitutes the Coulomb interaction included in Eq. (3.1). Due to its null size, the gravitational term in Eq. (3.1) would have to be removed if the quantum particle could not interact with itself. In this case, the kinetic energy term of Eq. (3. 1) would not have a local minimum which results in an infinitely spatial extended plane wave as the wavefunction for a free particle with mass. In contrast, $E_{Sch}(r)$ has a minimum when [4, 10]:

$$r = a_G = \frac{\hbar^2}{Gm^3} = l_P\left(\frac{m_P}{m}\right)^3, \text{with } l_P = \sqrt{\frac{\hbar G}{c^3}}, \text{and } m_P = \sqrt{\frac{\hbar c}{G}}. \qquad (3.10)$$

In Eq. (3.10), l_P and m_P are the Planck's length and mass, respectively.

At relativistic speeds, the Schrödinger-Newton equation (Eq. (3.8)) should be substituted by the GP-Newton equation:

$$i\hbar \frac{\partial}{\partial t}\psi(r,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2}\,\psi(r,t) - \,Gm^2 \int \frac{|\psi(r',t)|^2}{|r'-r|}dr'. \qquad (3.11)$$

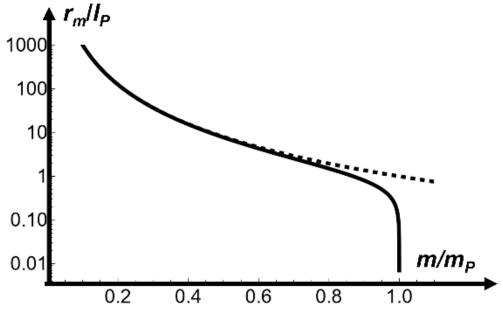

Fig. 3.2. (Discontinuous) Non-relativistic and (continuous) relativistic estimates of the radius (in Planck length units) of the common quantum field with $E_T = E + mc^2$ for a particle that interacts gravitationally with itself [4, 10]

And Eq (3.9) should be substituted by the following equation [4, 10]:

$$E_{GP}(r) \approx \frac{\hbar^2}{(\gamma+1)mr^2} - \frac{Gm^2}{r}\text{, with } \gamma = \sqrt{1 + \left(\frac{\lambda_C}{r}\right)^2}. \quad (3.12)$$

$E_{GP}(r)$ has a minimum when [4, 10]:

$$r = r_m = a_G\sqrt{1 - \left(\frac{\lambda_C}{a_G}\right)^2} = l_P\left(\frac{m_P}{m}\right)^3\sqrt{1 - \left(\frac{m}{m_P}\right)^4}. \quad (3.13)$$

As shown in Fig. 3.2, a notable consequence of combining quantum mechanics, the special theory of relativity, and Newtonian gravity is the existence of a critical mass $m_c = m_P$ above which the size of the particle becomes undefined. This critical mass could be interpreted as the frontier between the quantum and the classical matter world [4, 10]. This is because the quantum field, associated with a relativistic quantum particle with mass m in a common state, collapses to a point when $m = m_P$. When this happens the quantum particle is transformed into a classical particle. It should be noted that the Planck mass value ($m_P \approx 22$ μg) is quite small for having to consider the full complexity of quantum mechanics in daily life. In contrast, it is quite large when compared to molecular masses, and the quantum experiments that have been accomplished to date. Interestingly, biological cells, including human neurons, could still be quantum objects. In any event, the experimental confirmation or rejection of this hypothesis would have fundamental consequences for quantum mechanics and cosmology. In particular, the confirmation of the existence of m_c could mean that there is not a universal wavefunction, that the Schrödinger quantum cat cannot exist because his mass is much larger than m_P [4, 6] and that the macroscopic world that surrounds us is as classical as it seems to be for the same reason. Nevertheless, it is important to realize that huge classical bodies can be formed by numerous quantum particles.

Notably, if $m \approx m_P$, then the quantum particle moves at relativistic speeds, but its kinetic energy does not reach ultra-relativistic values larger than $2mc^2$. This is because if $m \approx m_P$, then $r \approx l_P$. Therefore, we could estimate K using Eq. (3.9):

$$K_{l_P} \approx \frac{\hbar^2}{2m_P {l_P}^2} = \frac{1}{2}\, mc^2. \tag{3.14}$$

This value is equal to Eq. (3.3) because the reduced Compton wavelength of a particle with mass $m = m_P$ is equal to the Planck length:

$$\lambda\!\!\!^{-}_C = \frac{\hbar}{m_P c} = \sqrt{\frac{\hbar G}{c^3}} = l_P\,. \tag{3.15}$$

In addition, if $m \approx m_P$, the absolute value of the gravitational potential energy of a particle with mass $m = m_P$ does not reach ultra-relativistic values larger than $2mc^2$:

$$\left|U_{l_P}\right| \approx \frac{Gm^2}{l_P} = mc^2. \tag{3.16}$$

Therefore, disregarding effects related to the creation of particle-antiparticle pairs in our simple approach is well justified.

Finally, it should be noted that there are two waves associated with a relativistic quantum particle. We have discussed above about the size of the quantum field corresponding to the common quantum states of a relativistic quantum particle that gravitationally interacts with itself. If the relativistic quantum particle were in an exotic state, then Eq. (3.12) should be substituted by the following equation:

$$E'_{GP}(r) \approx -\frac{\hbar^2}{(\gamma+1)mr^2} - \frac{Gm^2}{r}. \tag{3.17}$$

However, Eq. (3.17) does not have a local extremum because both the kinetic energy of the quantum particle in exotic states and its potential energy are negatives. This means there is no collapse of the exotic quantum field associated with a relativistic quantum particle for any mass value. Nevertheless, the collapse of one of the two quantum fields associated with a relativistic quantum particle with mass is enough for the inexistence of the relativistic quantum particle in a situation where the particle can only have a wave associated to it.

SECTION 3C. THE PLANCK CHARGE

It should be clear that the discussions presented in Sections 3.a and 3.b are also valid for antiparticles. This means that like Hydrogen-like atoms, anti-Hydrogen-like atoms could not be too heavy (Section 3.a). This also means that antimatter bodies with a mass larger than the Planck mass should be classical objects. In Sections 3.a and 3.b we explored two cases where attractive interaction exists.

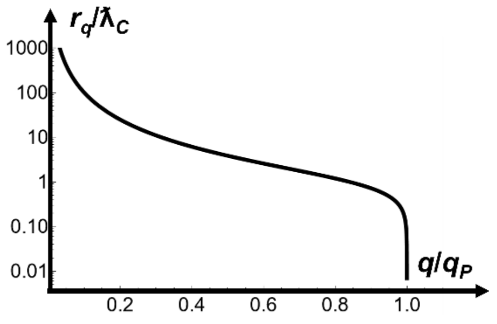

Fig. 3.3. Relativistic estimate of the radius (in Planck length units) of the exotic quantum field for an antiparticle with $E_{Ta} = E'_a - mc^2$ that interacts electrostatically with itself [4]

For a change, let us now start by referring to antiparticles and exotic antiparticle states. We will focus now on describing the hypothetical repulsive Coulombic interaction of an electrically charged antiparticle with itself. If the antiparticle is in an exotic state and has charge *q*, then Eq. (3.7) should be substituted by [4, 13]:

$$E'_{GPa}(r) \approx -\frac{\hbar^2}{(\gamma+1)mr^2} + \frac{q^2}{4\pi\epsilon_0 r}\text{ , with } \gamma = \sqrt{1+\left(\frac{ƛ_C}{r}\right)^2}\,. \tag{3.18}$$

$E'_{GPa}(r)$ has a maximum when [4, 13]:

$$r = r_q = ƛ_C \xi^{-2}\sqrt{1-\xi^4}\text{ , with } \xi = \frac{q}{q_P}\text{ , } q_P = \sqrt{4\pi\epsilon_0\hbar c}\,. \tag{3.19}$$

Therefore, $ƛ_C/\xi^{-2} \rightarrow ƛ_C$ (the reduced Compton wavelength), when $|q| \rightarrow q_P \approx 11e$ (the Planck charge). Moreover, $r \rightarrow 0$, when $|q| \rightarrow q_P$. Therefore, as shown in Fig. 3.3, a notable consequence of combining quantum mechanics, special theory of relativity, and repulsive Coulombic self-interactions is the existence of a critical charge, $|q_c| = q_P$. Above this charge's absolute value, the size of the antiparticle becomes undefined because the exotic quantum field collapses to a point. This critical charge could be interpreted as the frontier between the quantum and the classical antimatter world [4, 13].

There should be two waves associated with a relativistic quantum antiparticle. We have discussed above about the size of the quantum field corresponding to the exotic quantum states of a relativistic quantum antiparticle that electrically interacts with itself. If the antiparticle were in a common state, then Eq. (3.18) should be substituted by the following equation:

$$E_{GPa}(r) \approx \frac{\hbar^2}{(\gamma+1)mr^2} + \frac{q^2}{4\pi\epsilon_0 r}\text{ , with } \gamma = \sqrt{1+\left(\frac{ƛ_C}{r}\right)^2}\,. \tag{3.20}$$

However, Eq. (3.20) does not have a local extremum because both the kinetic energy of the antiparticle in common states and its potential energy are positive. This means there is no collapse of the quantum field associated with the common states of the antiparticle for any value of its charge q. Nevertheless, the collapse of one of the two quantum fields associated with a relativistic quantum antiparticle is enough for the inexistence of the antiparticle in a situation where the antiparticle can only have a wave associated with it. This means that quantum antiparticles with the modulus of its electric charge ($|q|$) larger than the Plank charge should not exist. In fact, no quantum antiparticle with $|q| > q_P$ has ever been observed.

Notably, if $|q| \approx q_P$, then the quantum antiparticle moves at relativistic speeds, but neither the kinetic energy of the antiparticle nor the absolute value of its potential energy (U) reaches ultra-relativistic values larger than $2mc^2$. This is because if $|q| \approx q_P$, then $r \approx ƛ_C$. Therefore, we could estimate the absolute value of $|K'|$ and U in the following way:

$$\left|K'_{ƛ_C}\right| \approx \frac{\hbar^2}{2mƛ_C{}^2} = \frac{1}{2}\, mc^2, \text{and } U_{ƛ} \approx \frac{q^2}{4\pi\epsilon_0 ƛ} = mc^2. \tag{3.21}$$

Therefore again, disregarding the effects related to the creation of particle-antiparticle pairs in our simple approach is well justified.

Chapter 4

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Braking the Particle-antiparticle Symmetry

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At this point, we can focus our attention on the evident asymmetry between matter and antimatter that exists in the world surrounding us. As stated in Section 3.3, particles and antiparticles are formed in pairs, so therefore, the number of particles in the Universe should be equal to the number of antiparticles [1-4]. The fact that we seem to live in a Universe where there are many more particles than antiparticles is an unsolved mystery [3]. In addition, no one knows why we live in a world formed almost exclusively by atoms and molecules made of matter. No antimatter life seems to exist in the Universe. Most physicists believe that the reason for this huge discrepancy between theory and reality is related to some type of unknown asymmetric event that occurred at the beginning of the Universe [3]. But nobody knows for sure why and how the observed asymmetry occurs.

It should be emphasized that everything discussed until this point in this monograph is strictly particle-antiparticle symmetric. Thus, coincides in this aspect with the big picture coming from the standard model of particle physics [1]. However, the interaction of a fundamental particle with itself is not included in the standard model of particle physics. This gives us the theoretical opportunity to propose a way of breaking the theoretical matter-antimatter symmetry by adequately extending the standard model of particle physics. This extension should simultaneously break the theoretical matter-antimatter symmetry and include the interaction of a quantum particle with itself.

SECTION 4A. SHORTAGE OF ANTIMATTER ELEMENTS

By hypothesizing that a quantum particle could interact electrostatically with itself, we could find a simple explanation for the observed shortage of antimatter elements (Z < 2-3) when compared with the relatively larger number of elements in the Periodic table (Z < 120). We should add that the symmetry breakdown requires that a quantum particle interacts with itself, but in a different manner than the corresponding antiparticle does. In Section 3, it was proposed that the antiparticle self-repulsive Coulombic interaction results in the validity of Eqs. (3.20) and (3.18). This corresponds to the validity of the following nonlinear relativistic wave equations, respectively [13]:

$$i\hbar\frac{\partial}{\partial t}\psi_a(r,t) = -\frac{\hbar^2}{2\mu_a}\frac{\partial^2}{\partial r^2}\,\psi_a(r,t) + \frac{q^2}{4\pi\epsilon_0}\int\frac{|\psi_a(r',t)|^2}{|r'-r|}dr'. \tag{4.1}$$

and

$$i\hbar\frac{\partial}{\partial t}\Omega_a(r,t) = -\frac{\hbar^2}{2\mu_a'}\frac{\partial^2}{\partial r^2}\,\Omega_a(r,t) + \frac{q^2}{4\pi\epsilon_0}\int\frac{|\Omega_a(r',t)|^2}{|r'-r|}dr'. \tag{4.2}$$

If the charged antiparticle is in a common state, Eq. (4.1) is valid. If the charged antiparticle is in an exotic state, Eq. (4.2) is valid. In contrast, the matter-antimatter symmetry can be broken by proposing, first, that the corresponding particle with charge – q electrostatically interacts with itself and, second, that the following wave equations are valid for the particle [13]:

$$i\hbar\frac{\partial}{\partial t}\psi(r,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial r^2}\,\psi(r,t) + \frac{q^2}{4\pi\epsilon_0}\int\frac{|\psi(r',t)|^2}{|r'-r|}dr'. \tag{4.3}$$

and

$$i\hbar\frac{\partial}{\partial t}\Omega(r,t) = -\frac{\hbar^2}{2\mu'}\frac{\partial^2}{\partial r^2}\,\Omega(r,t) - \frac{q^2}{4\pi\epsilon_0}\int\frac{|\Omega(r',t)|^2}{|r'-r|}dr'\,. \tag{4.4}$$

If the charged particle is in a common state, Eq. (4.3) is valid. If the charged particle is in an exotic state, Eq. (4.4) is valid. Note that the matter-antimatter symmetry is broken because Eqs. (4.2) and (4.4), the wave equations for the exotic states, are not equal. Solving the wave equations (Eqs. (4.1) to (4.4)) may present notable mathematical difficulties due to their nonlinear character. However, as shown in Section 3.c., we could have a crude estimate of the physical reality corresponding to them by solving Eqs. (3.18), (3.20) and the following equation:

$$E'_{GP}(r) \approx -\frac{\hbar^2}{(\gamma+1)mr^2} - \frac{q^2}{4\pi\epsilon_0 r}\,, \quad \text{con }\gamma = \sqrt{1+\left(\frac{\lambda_C}{r}\right)^2}\,. \tag{4.5}$$

In contrast with the equation corresponding to the exotic state of the antiparticle (Eq. (3.18), the equation corresponding to the exotic state of the particle (Eq. (4.5) does not have a local minimum. For this reason, the hypothesized Coulombic self-interaction breaks the particle-antiparticle symmetry. Note that if the relativistic quantum particle is in a common state, then Eq. (3.20) should also be valid for the relativistic quantum particle.

Consequently, only the exotic quantum field associated with the antiparticle could collapse to a point due to the Coulombic interaction of the antiparticle with itself. This is a theoretical proposal that is outside of the standard model of particle physics. This is because it supposes the interaction of every relativistic quantum particle and antiparticle with itself. This proposal does not affect the standard model of particle physics in its scope of applications but extends the range of applications of relativistic quantum mechanics. Moreover, as will be shown below, this theoretical proposal is strongly validated by the compelling match between its predictions and the physical reality surrounding us.

First, we could easily explain the observed shortage of antimatter chemical elements when compared with the relative abundance of chemical elements in the Periodic table. The stability of atoms is a consequence of quantum mechanics. In nonrelativistic quantum mechanics, it is the existence of a wave associated with a quantum particle that makes atoms stable. In relativistic quantum mechanics, there are two waves associated with a relativistic quantum particle. Atoms and antiatoms are stable if and only if there are two waves associated with the electron and positron cloud surrounding the nucleus, respectively.

A stable Hydrogen-like atom requires a relativistic quantum electron attracted by the nucleus. Two waves should be associated with the electron. However, as discussed in Section 3.a, if $Z > 137$, the common wave associated with the electron collapses to a point. This theoretical prediction matches the no existence of atoms with $Z > 137$. This is also valid for anti-Hydrogen-like antiatoms formed by an antiproton and a positron. This is because Eqs. (3.4) and (3.7) are equally valid for Hydrogen-like atoms and anti-Hydrogen-like antiatoms.

In contrast, there is an additional theoretical limitation only for antiatoms. A stable antiatom requires a relativistic quantum cloud of positrons attracted by the nucleus. Two waves should be associated with the cloud of positrons. However, as discussed in Section 3.c, if $Ze > q_P \approx 11e$, the exotic wave associated to the cloud of positrons collapses to a point. This theoretical prediction matches the no existence of antiatoms with $Z > 11$. This constitutes an additional and stronger limitation to the atomic number of possible antiatoms. However, due to Eq. (4.5), the exotic wave associated with a cloud of electrons never collapses to a point. Consequently, there are matter atoms with $11 < Z < 137$. This explains the observed shortage of antimatter elements when compared with the relatively larger number of elements in the Periodic Table [4, 13].

SECTION 4B. BIOLOGICAL ANTIMATTER CAN NOT EXIST

The prediction about the impossibility of the existence of atoms with $Z > 137$ is a correct but approximated prediction. No atom with $Z > 118$ has ever been observed. Similarly, the prediction about the impossibility of the existence of antiatoms with $Z > 11$ is a correct but approximated prediction. No antiatom with $Z > 2\text{-}3$ has ever been observed. We can then argue that Carbon antiatoms with $Z = 6$ do not exist because if $Z = 6$ the quantum field associated with the exotic state of a cloud of 6 positrons collapses to near a point. The impossibility of the existence of Carbon antiatoms explains the reality surrounding us. We are surrounded by biological matter, but biological antimatter has never been observed [4,13].

SECTION 4C. PRIMORDIAL BLACK HOLES AND ANTIMATTER ELECTRICAL SINKS

Previously, we have argued that there is no wave associated with a classical particle but there are two waves associated with a relativistic quantum article. If one of the waves collapses to a point, then the quantum particle transforms into a classical particle. Classical particles cannot form stable atoms.

There is another hypothetical situation where the collapse of the quantum field to a point could be relevant [4, 13]. It has been hypothesized the possible existence of primordial black holes with a relatively small mass. Primordial black holes may have been created around 13 billion years ago, at the beginning of our universe. Mass fluctuations with $m > m_P$ could have produced their formation [12]. As discussed above, these hypothetical mass fluctuations may have formed primordial relativistic quantum objects. If their masses were larger than Planck's mass, then the collapse to points of their common quantum fields (Fig. 3.2) may have created primordial black holes. At present, there is no observational evidence of the existence of primordial black holes. Nevertheless, the possible existence of these small mass black holes is a research topic of great interest.

Similarly, primordial antimatter electric sinks may have been created around 13 billion years ago, at the beginning of our universe [4, 13]. Antimatter charge fluctuations with $|q| > q_P$ could have produced their formation. As discussed above, these hypothetical antimatter charge fluctuations may have formed primordial relativistic quantum objects. If their charges were larger than Planck's charge, then the collapse to points of their exotic quantum fields (Fig. 3.3) may have created primordial antimatter electric sinks. This may explain the existence of an excess of charged matter in the rest of the universe [4, 13].

CONCLUSION

In this work, it was argued that current relativistic quantum mechanics should be expanded by including the interaction of each quantum particle and antiparticle with itself. This work discusses how we could explain the everyday experience of the absence of biological antimatter in our world. It is shown that this could be achieved by maintaining the validity of relativistic quantum mechanics, but adopting the idea that both an electrically charged particle with mass and the corresponding antiparticle could interact electrically with itself.

For simplicity, the discussions were based on the solutions of the Grave de Peralta equation for the infinity well. This is the simplest model for a spatially localized relativistic quantum particle with mass. As discussed in Chapter 2, the Grave de Peralta equation is a relativistic but Schrödinger-like equation. Therefore, it can be solved like the Schrödinger equation is solved. The well-known solution of the Schrödinger equation for a quantum particle with mass, which is confined in an infinite one-dimensional well, was presented in Chapter 1. The corresponding problem for the relativistic Grave de Peralta equation was solved in Chapter 2.

A semiquantitative discussion of the consequences of adding the interaction of the quantum particle with itself was presented in Chapter 3. This includes the determination of the size of Hydrogen-like atoms, the possible existence of a frontier between the quantum and the classical world, and the impossibility of the existence of quantum antiparticles with the absolute value of the electrical charge larger than the Plank charge. Finally, in Chapter 4, the matter-antimatter symmetry was theoretically broken by postulating that a particle electrically interacts with itself in a different way than the corresponding antiparticle interacts with itself. It was then presented a surprising explanation of this everyday experience: we are surrounded by living beings made of matter, but no living beings made of antimatter have ever been observed.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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INDEX

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